SOLUTIONS TO THE QUIZ

Problem 1

The daily interest rate is $\frac{0.10}{365}$ = 0.0002739726.

Let T be the number of days needed for the account value to exceed $1,200.

Account value after T days = $1000 \cdot (1 + 0.0002739726)^T$. So,

$$1000 \cdot (1.0002739726)^T > 1200$$

$$(1.0002739726)^T > 1.2$$

$$T \cdot \ln(1.0002739726) > \ln(1.2)$$

$$T > \frac{\ln 1.2}{\ln 1.0002739726} = 665.56485$$

So, at least 666 days are needed for the account value to exceed $1,200.

Note that due to daily compounding, the required time is less than 2 years by about 2 months!

Problem 2

Number of ways of selecting 5 widgets with 2 defectives from available set of widgets

= (# of ways of selecting 2 defectives out of 25) · (# of ways of selecting 3 OK out of 75)

= $^{25}C_2 \cdot {}^{75}C_3$

$= \frac{25!}{2! \cdot 23!} \cdot \frac{75!}{3! \cdot 72!}$

$= \frac{(25)\cdot(24)}{(2)\cdot(1)} \cdot \frac{(75)\cdot(74)\cdot(73)}{(3)\cdot(2)\cdot(1)}$

= (300) · (67,525) = 20,257,500

Number of ways of selecting 5 widgets from available set of widgets without regard to defectives

= # of ways of selecting 5 widgets out of 100

$= {}^{100}C_5 = \frac{100!}{5! \cdot 95!} = \frac{(100)\cdot(99)\cdot(98)\cdot(97)\cdot(96)}{(5)\cdot(4)\cdot(3)\cdot(2)\cdot(1)} = 75{,}287{,}520$

Probability of obtaining 2 defectives in 5 randomly selected widgets

= (# of ways of selecting 5 widgets with 2 defectives)/(# of ways of selecting 5 widgets without regard to defectives)

$= \frac{20{,}257{,}500}{75{,}287{,}500}$

= 0.2690685 or 26.90685%

Problem 3

Let Π denote the monopolist's profit.

$$\begin{aligned}\Pi &= \text{Revenue} - \text{Cost}\\ &= P\cdot Q - [750 + 8Q]\\ &= P\cdot(100 - P) - [750 + 8\cdot(100 - P)]\\ &= -P^2 + 108P - 1550\end{aligned}$$

Taking derivatives with respect to P of both sides and setting equal to zero:

$$\frac{d\Pi}{dP} = -2P + 108 = 0$$

And so, optimal $P = 108/2 = \$54$.

Problem 4

Let D denote the event that a randomly selected widget is defective.
Let A denote the event that a randomly selected widget is produced by Factory A.
Let B denote the event that a randomly selected widget is produced by Factory B.
Let $\Pr(X \mid Y)$ denote the conditional probability of event X given event Y.

We are told that $\Pr(A) = 0.20$; $\Pr(B) = 0.80$; $\Pr(D \mid A) = 0.04$ and $\Pr(D \mid B) = 0.01$
We are asked to find $\Pr(A \mid D)$.

$$\begin{aligned}\Pr(A \text{ and } D) &= \Pr(D \mid A) \cdot \Pr(A)\\ &= (0.04) \cdot (0.20) = 0.008\end{aligned}$$

$$\begin{aligned}\Pr(D) &= \Pr(D \text{ and } A) + \Pr(D \text{ and } B)\\ &= \Pr(D \mid A) \cdot \Pr(A) + \Pr(D \mid B) \cdot \Pr(B)\\ &= (0.04)\cdot(0.20) + (0.01)\cdot(0.80) = 0.008 + 0.008 = 0.016\end{aligned}$$

Finally,
$$\begin{aligned}\Pr(A \mid D) &= \Pr(A \text{ and } D) / \Pr(D)\\ &= 0.008 / 0.016\\ &= 50\%\end{aligned}$$

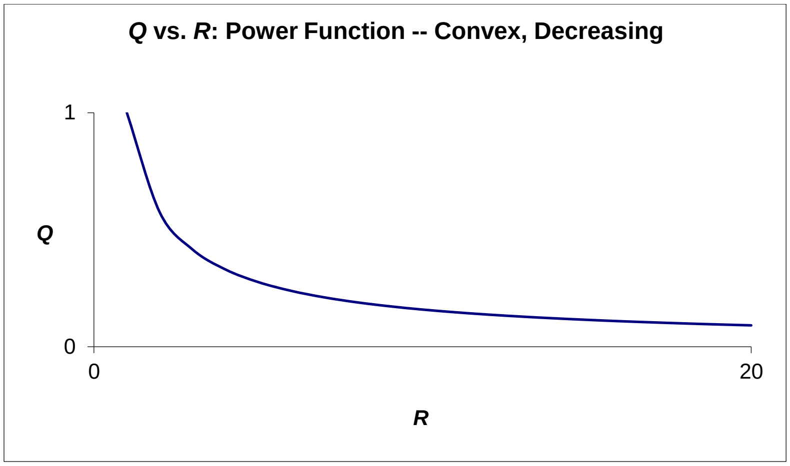
Problem 5

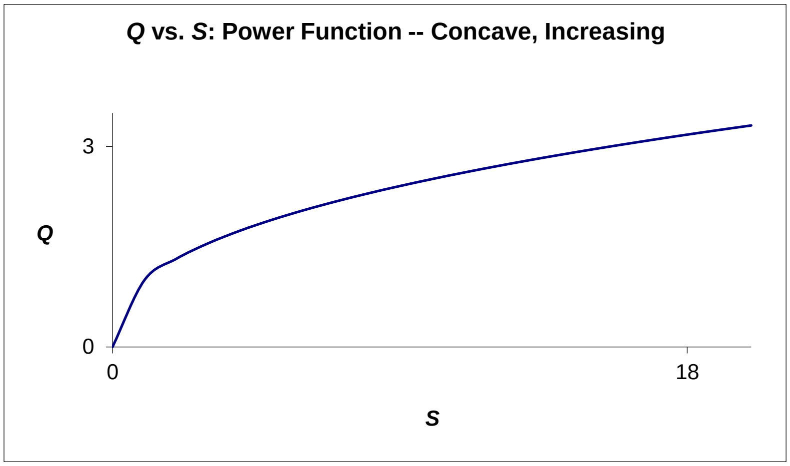
Let r denote the constant annual percentage increase that would have effectively produced the same price at the end of three years. Then,

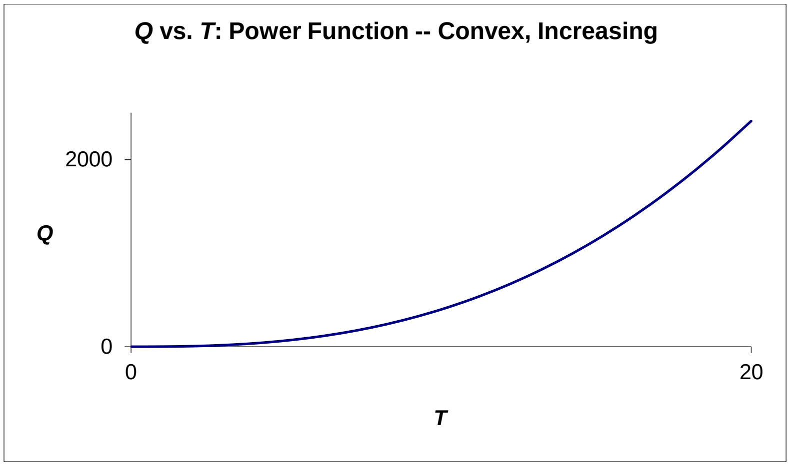
$$\begin{aligned}(1 + r)^3 &= (1 + 0.40)\cdot(1 + 0.01)\cdot(1 + 0.10) = (1.40)\cdot(1.01)\cdot(1.10) = 1.5554\\ 1 + r &= (1.5554)^{1/3} = 1.1586369\\ r &= 0.1586369 = 15.86369\%\end{aligned}$$

Note that this is quite different from the arithmetic average of the three annual increases of 10%, 1% and 40% (which is 17%).

Problem 6


Q vs. R: Power Function -- Convex, Decreasing
1
Q
0
0
20
R


Q vs. S: Power Function -- Concave, Increasing
3
Q
0
0
18
S


Q vs. T: Power Function -- Convex, Increasing
2000
Q
0
0
20
T

Problem 6 (continued)